Effect of Ion Engine Exhaust on the Propagation of Electromagnetic Waves

A. R. M. Rashad*
University of Detroit, Detroit, Mich.

A theory is presented concerning the effect of the ion engine exhaust on the propagation, interaction, and scattering of incident electromagnetic waves originating from a nearby dipole antenna. The exhaust is represented as a cylindrical beam. Two cases of interest are studied: that of an exhaust beam in an axial d.c. magnetic field $B_z = 0$, and that of a finite value B_z, respectively. The electromagnetic field equations are obtained for transverse magnetic (*TM*) modes in the first case and for transverse electric (*TE*) modes in the second case. The analysis considers also the changes in the beam temperature due to its interaction with the surrounding medium. The fluctuations in the coefficients of the tensor dielectric constant [ε_b] due to the temperature changes and their effect on the electromagnetic field expressions also are investigated.

Introduction

IN a recent paper, Parzen[1] analyzed theoretically the effect of the ion engine exhaust on telemetry systems. Recently, Rashad[2, 3] studied the effect of the engine exhaust on the radiation field of a nearby dipole antenna. The model used in Ref. 2 is an infinite plasma slab that represents the exhaust conditions, as was suggested by Rosenbluth et al.[4] and Seitz et al.[5] Rashad[3] used a cylindrical configuration to represent the exhaust conditions. However, in all of these previous studies, the effect of the axial magnetic field has been completely neglected, and therefore the exhaust beam has been represented by a dielectric medium with a scalar dielectric constant.

In the present study, the effect of the axial magnetic field is taken into consideration. It was shown by Mickelsen et al.[6] and Reader[7,8] that under actual operating conditions of the engine, an axial magnetic field B_z of a finite value is applied to the beam. Therefore, the exhaust medium in this case should be considered as an anisotropic medium with a tensor dielectric constant [$\boldsymbol{\varepsilon}_b$].

In the following paper, the exhaust beam is represented as a cylindrical plasma beam. The source of the electromagnetic waves considered is a nearby dipole antenna. The equations of propagation of the incident waves and their scattering from the exhaust beam are investigated in detail. The paper is divided into three parts, each relating to the magnetic field value. The first part considers the case of the exhaust under zero applied axial magnetic field. For *TM* modes, the dipole current is transformed by a Fourier series to an equivalent cosinusoidally distributed current surrounding the beam. A radiation pattern function $F(\phi)$ is calculated which determines the azimuthal beam synthesis for the dipole source. The integrals in the $F(\phi)$ expression are evaluated by the saddle point of integration method. In Sec. II of the paper, the effect of the axial magnetic field is considered. The exhaust beam is represented as an anistropic homogeneous plasma medium with a tensor dielectric constant [$\boldsymbol{\varepsilon}_b$]. The coefficients of [$\boldsymbol{\varepsilon}_b$] are functions of the beam temperature, axial velocity, density of electrons, collision frequency, and applied magnetic field. The equations of propagation and scattering of incident *TE* electromagnetic waves on the beam are obtained. The solution of these equations expresses the **E** and **H** fields in terms of Bessel and Hankel functions. Methods for the numerical evaluation of the resulting field expressions are pointed out. Section III of this paper deals with the effect of the exhaust beam temperature change on the resultant electromagnetic field expressions developed in Sec. II. As Shapiro[9, 10] has shown, the scattering of a plasma beam within a surrounding medium changes its temperature anisotropically in case $B_z \neq 0$ and isotropically in case $B_z = 0$. The corresponding fluctuations in the electromagnetic field expressions due to the exhaust temperature change are investigated in this part of the study.

Presented as Preprint 65-186 at the AIAA 6th Solid Propellant Rocket Conference, Washington, D. C., February 1–3, 1965; revision received April 21, 1965.

* Associate Professor of Electrical Engineering.

I. Exhaust Beam Alone

In this part, the engine exhaust will be represented as a uniform longitudinal cylindrical beam of radius a, coaxial with the z axis, and having an average axial velocity v_0 as shown in Fig. 1. The axial magnetic field B_z is assumed to be zero in this case. The source of electromagnetic waves is taken as a nearby dipole antenna oriented parallel to the z axis, and located at a distance $r = b$ and $\phi = 0$ from the beam axis. The dipole will be considered as a linear current source I. For cylindrical coordinates (r, ϕ, z), this linear current source can be transformed to an equivalent cosinusoidally distributed current sheet of radius b coaxial with the exhaust beam by the following Fourier Series:

$$\frac{I\delta(\phi)}{b} = \sum_{m=0}^{\infty} K_z = \sum_{m=0}^{\infty} K_m \cos m\phi \qquad (1)$$

where K_z is the equivalent current density representation of the current source, $K_m = \epsilon_m(I/2\pi b)$ is the amplitudes of the equivalent cosinusoidally distributed surface current densities, $\delta(\phi)$ is a delta function, and ϵ_m is the Neumann number, given by $\epsilon_0 = 1$ and $\epsilon_m = 2$ for m $\neq 0$, provided that m is an integer, so that K_z will be a single valued function of ϕ. The field equations, given by Rashad,[3] read as follows:

Region I: $r \leqslant a$

$$E_{z_1} = \sum_{m=0}^{\infty} A_m J_m(Kr) \cdot \cos m\phi$$

$$H_{\phi_1} = \sum_{m=0}^{\infty} \frac{A_m}{j\zeta_b} J_m{}'(Kr) \cdot \cos m\phi \qquad (2)$$

Region II: $a \leqslant r \leqslant b$

$$E_{z_2} = \sum_{m=0}^{\infty} [B_m J_m(Kr) + C_m H_m{}^{(2)}(Kr)] \cos m\phi$$

$$H_{\phi_2} = \sum_{m=0}^{\infty} [B_m J_m{}'(Kr) + C_m H_m{}'^{(2)}(Kr)] \cdot \frac{\cos m\phi}{j\zeta_s} \qquad (3)$$

Region III: $b \leqslant r$

$$E_{z3} = \sum_{m=0}^{\infty} D_m H_m^{(2)}(Kr) \cdot \cos m\phi$$

$$H_{\phi 3} = \sum_{m=0}^{\infty} \frac{D_m}{j\zeta_s} H_m'^{(2)}(Kr) \cdot \cos m\phi \qquad (4)$$

where the time dependence $e^{j\omega t}$ has been omitted in each term, J_m and $H_m^{(2)}$ are Bessel and Hankel functions, A_m, B_m, C_m, and D_m are constants to be determined from the boundary conditions, ζ_b is the surface impedance of the exhaust beam medium and ζ_s is the intrinsic impedance of the medium surrounding the exhaust beam.

Solution of Eqs. (2–4) is given by Rashad[3] to read

$$E_z = \sum_{m=0}^{\infty} \epsilon_m \frac{\omega I}{4} [J_m(Kb) + \alpha_m H_m^{(2)}(Kb)] H_m^{(2)}(Kr) \cdot \cos m\phi \qquad (5)$$

where

$$\alpha_m = -[J_m'(Ka) + GJ_m(Ka)]/[H_m'^{(2)}(Ka) + GH_m^{(2)}(Ka)]$$

$$G = -j(\zeta_b/\zeta_s) \cong -j(\epsilon_s/\epsilon_b)^{1/2}$$

where ϵ_b and ϵ_s are the dielectric constants of the exhaust beam medium and the medium surrounding it, respectively.

Since $B_z = 0$, the exhaust can be represented as a homogeneous plasma medium. Its equivalent dielectric constant was given by Marcuvitz[11] and Parzen,[1] to read as

$$\epsilon_b = \epsilon_0\{1 - [\omega_p^2/(\omega - Kv_0)^2]\} \qquad (6)$$

where ω_p is the plasma frequency in the base medium, and ϵ_0 is the permittivity of free space. Equation (6) is valid also, in case the exhaust is represented as an electron beam alone, as an ion beam alone or as a neutralized beam also, provided that the proper ω_p is used in this equation. If the drifting effect of the beam within its surrounding medium is considered, the dielectric constant as given by Eq. (6) becomes

$$\epsilon_b = \epsilon_0\{1 - [\omega_p^2/(\omega - Kv_0)^2] - (\omega_p'^2/\omega^2)\} \qquad (6')$$

where ω_p' is the plasma frequency in the medium surrounding the exhaust beam.

For the far field calculations, $H_m^{(2)}(Kr)$ in Eq. (5) is replaced by its asymptotic value given by Magnus[12]:

$$H_m^{(2)}(Kr) \cong (2/\pi Kr)^{1/2} \cdot \exp\{-j[Kr - \tfrac{1}{2}m\pi - (\pi/4)]\}$$

Therefore

$$E_z = [\omega I/(8\pi Kr)^{1/2}] \cdot \exp\{-j[Kr - (\pi/4)]\} \cdot F(\phi) \quad (7)$$

where $F(\phi)$ is a radiation pattern function that determines the azimuthal beam synthesis for the dipole source, and is given by

$$F(\phi) = \sum_{m=0}^{\infty} \epsilon_m [J_m(Kb) + \alpha_m H_m^{(2)}(Kb)] \cdot \cos m\phi \cdot \exp\left(\frac{jm\pi}{2}\right) \qquad (8)$$

Since the field is usually evaluated at large distances r compared to the radii a and b, the evaluation of the function $F(\phi)$ as given by Eq. (8) is greatly simplified by considering $a \cong b$. In this case,

$$F(\phi) \cong \frac{2}{j\pi x} \sum_{m=0}^{\infty} \frac{\epsilon_m \cdot \cos m\phi \cdot \exp(jm\pi/2)}{H_m'^{(2)}(x) + GH_m^{(2)}(x)} \qquad (9)$$

where $x = Ka$. If x is not excessively large, the $F(\phi)$ expression given by Eq. (9) can be evaluated from the results of Wait,[13,14] to read as follows:

$$F(\phi) \cong e^{-jKa\theta} \cdot g[\theta(\tfrac{1}{2}Ka)^{1/3}] + e^{-jKa(\pi-\theta)} \cdot g[(\pi - \theta)(\tfrac{1}{2}Ka)^{1/3}] \quad (10)$$

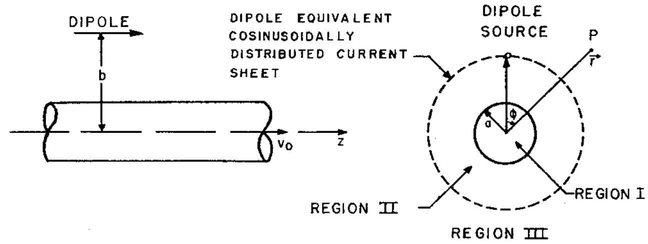

Fig. 1 Engine exhaust beam and dipole antenna configuration.

which is valid for $\pi/2 \geqslant |\theta| \geqslant -\Delta\theta$, provided that $(\Delta\theta)^3 \ll 1$, and where

$$\theta = \phi - (\pi/2)$$

$$g(X) = \frac{1}{\pi} \int_\Gamma \frac{e^{-jXt}}{W_1'(t) - qW_1(t)} dt$$

Where $W_1(t)$ is an airy integral

$$q = G(\tfrac{1}{2}Ka)^{1/3} = -j(\tfrac{1}{2}Ka)^{1/3} \cdot (\zeta_b/\zeta_s)$$

and Γ is an integration contour that runs from $\infty e^{-j(2\pi/3)}$ to 0, and then out along the real axis to ∞. The integral $g(X)$ can be evaluated by the saddle point of integration method, to give the following expression for small values of θ:

$$F(\phi) \cong e^{-jKa\theta} \cdot e^{jKa \cdot (\theta^3/3!)} \cdot \{2\theta/[\theta - (\zeta_b/\zeta_s)]\} \qquad (11)$$

If the medium surrounding the exhaust beam is taken as free space, and the drifting effect of the beam is neglected, then from Eqs. (6, 7, and 11), the resultant field E_z can be given by

$$E_z \cong \frac{\omega I}{(8\pi Kr)^{1/2}} \cdot \left[\frac{2\theta}{\theta - \{(\omega - Kv_0)/[(\omega - Kv_0)^2 - \omega_p^2]^{1/2}\}}\right] \cdot e^{-j[Kr-(\pi/4)]} \cdot e^{-jKa[\theta - (\theta^3/3!)]} \qquad (12)$$

II. Exhaust Beam in an Axial Magnetic Field

In this case, the cylindrical exhaust beam is considered as an anisotropic homogeneous plasma medium. This anisotropy effect is due to the applied axial magnetic field $\mathbf{B}_0$ directed along the z axis. The dielectric constant of the beam is given by

$$[\boldsymbol{\varepsilon}_b] = \begin{vmatrix} \epsilon_1 & j\epsilon' & 0 \\ -j\epsilon' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{vmatrix} \qquad (13)$$

where ϵ_1, ϵ', and ϵ_3 are complex functions that depend on the angular frequency ω, the density of electrons, the collision frequency, the applied magnetic field $\mathbf{B}_0$, the beam temperature, and the axial velocity. The coefficients of the tensor $[\boldsymbol{\varepsilon}_b]$ have been calculated accurately by several investigators, e.g., Allis et al.[15] and Stix.[16] Following Stix,[16] the Boltzmann-Maxwell set of equations can be solved together to obtain a mobility tensor. The final expressions for $[\boldsymbol{\varepsilon}_b]$ are found in this reference, and since they are quite complex, they will not be included in this paper. The inverse of the tensor $[\boldsymbol{\varepsilon}_b]$ is given by

$$[\boldsymbol{\varepsilon}_b]^{-1} = \begin{vmatrix} \Delta_1 & j\Delta_2 & 0 \\ -j\Delta_2 & \Delta_1 & 0 \\ 0 & 0 & \Delta_3 \end{vmatrix} \qquad (14)$$

where

$$\Delta_1 = \epsilon_1/(\epsilon_1^2 - \epsilon'^2)$$

$$\Delta_2 = -\epsilon'/(\epsilon_1^2 - \epsilon'^2)$$

$$\Delta_3 = 1/\epsilon_3$$

The configuration of the exhaust beam, the axial magnetic field B_z, and the source of electromagnetic waves are shown in Fig. 2. Assuming no variation in the z direction, therefore, the field equations are given by the following:

Region I: $a \leqslant r$ (except at $r = b$ and $\phi = 0$)

$$\begin{aligned}(\nabla_t^2 + K^2)H_z &= 0 \text{ for } TE \text{ modes}\\ (\nabla_t^2 + K^2)E_z &= 0 \text{ for } TM \text{ modes}\end{aligned} \tag{15}$$

Region II: $r \leqslant a$

$$\begin{aligned}(\nabla_t^2 + \gamma_1^2)H_z &= 0 \text{ for } TE \text{ modes}\\ (\nabla_t^2 + \gamma_2^2)E_z &= 0 \text{ for } TM \text{ modes}\end{aligned} \tag{16}$$

where

$$\gamma_1^2 = K^2/\Delta_1 \qquad \gamma_2^2 = K^2/\Delta_3 \qquad K^2 = \omega^2\mu_0\epsilon_0$$

(if the medium surrounding the beam is considered as free space) and ∇_t^2 is the Laplacian operator in the transverse direction.

If TE modes are considered in this case, the source of electromagnetic waves can be taken as a magnetic current filament of strength M located at a distance $r' = b$ and $\phi = 0$ from the beam axis, as shown in Fig. 2. The incident field (region I) is given by Harrington[17]:

$$H_z^i = (K^2M/4\omega\mu_0)H_0^{(2)}(K|\mathbf{r} - \mathbf{r}'|)$$

Using the addition theorem of Hankel functions, H_z^i can be written as

$$\begin{aligned}H_z^i &= \frac{K^2M}{4\omega\mu_0}\sum_{m=-\infty}^{\infty} H_m^{(2)}(Kb)J_m(Kr)\cdot e^{jm\phi} \quad \text{for } r < b\\ &= \frac{K^2M}{4\omega\mu_0}\sum_{m=-\infty}^{\infty} J_m(Kb)H_m^{(2)}(Kr)\cdot e^{jm\phi} \quad \text{for } b < r\end{aligned} \tag{17}$$

The scattered field is

$$H_z^s = \frac{K^2M}{4\omega\mu_0}\sum_{m=-\infty}^{\infty} A_mH_m^{(2)}(Kb)H_m^{(2)}(Kr)\cdot e^{jm\phi} \tag{18}$$

where A_m is a constant to be determined from the proper boundary conditions. The ϕ component of the electric field $\mathbf{E}$ is evaluated from Maxwell equations and Eqs. (17) and (18) to give the following:

$$\begin{aligned}E_\phi^i &= \frac{jKM}{4}\sum_{m=-\infty}^{\infty} H_m^{(2)}(Kb)J_m'(Kr)\cdot e^{jm\phi}\\ E_\phi^s &= \frac{jKM}{4}\sum_{m=-\infty}^{\infty} A_mH_m^{(2)}(Kb)H_m'^{(2)}(Kr)\cdot e^{jm\phi}\end{aligned} \tag{19}$$

Within the beam (region II: $0 < r < a$) the field equations are obtained in a similar way to give the following:

$$H_z = \frac{K^2M}{4\omega\mu_0}\sum_{m=-\infty}^{\infty} [B_mJ_m(\gamma_1r) + C_mY_m(\gamma_1r)]H_m^{(2)}(Kb)\cdot e^{jm\phi}$$

$$E_\phi = \frac{jM}{4}\sum_{m=-\infty}^{\infty}\left[\frac{-m\Delta_2}{r}\{B_mJ_m(\gamma_1r) + C_mY_m(\gamma_1r)\} + \Delta_1\gamma_1\{B_mJ_m'(\gamma_1r) + C_mY_m'(\gamma_1r)\}\right]\cdot H_m^{(2)}(Kb)\cdot e^{jm\phi} \tag{20}$$

where Y_m is a Bessel function of the second kind of order m, and B_m, and C_m are constants to be determined from the proper boundary conditions. Solution of Eqs. (15–20) is straightforward and similar to the procedure used in Sec. I of this paper. For far field calculations ($b < r$) the resultant field expressions are

$$\begin{aligned}H_{z\text{total}} &= \frac{K^2M}{4\omega\mu_0}\sum_{m=-\infty}^{\infty} [J_m(Kb) + A_mH_m^{(2)}(Kb)]H_m^{(2)}(Kr)\cdot e^{jm\phi}\\ E_{\phi\text{total}} &= \frac{jKM}{4}\sum_{m=-\infty}^{\infty} [J_m'(Kr) + A_mH_m'^{(2)}(Kr)]H_m^{(2)}(Kb)\cdot e^{jm\phi}\end{aligned} \tag{21}$$

where

$$A_m = \frac{KJ_m(\gamma_1a)J_m'(Ka) + (m/a)\Delta_2J_m(\gamma_1a)J_m(Ka) - \gamma_1\Delta_1J_m(Ka)J_m'(\gamma_1a)}{KJ_m(\gamma_1a)H_m'^{(2)}(Ka) + (m/a)\Delta_2J_m(\gamma_1a)H_m^{(2)}(Ka) - \gamma_1\Delta_1H_m^{(2)}(Ka)J_m'(\gamma_1a)}$$

It is clear that the field expressions given by Eq. (21) are not easy for numerical calculations. It is suggested that a digital computer be used in the evaluation of the total $\mathbf{E}$ and $\mathbf{H}$ fields. If Eqs. (7, 8, 12, and 21) are compared, the effect of the axial magnetic field $\mathbf{B}_0$ on the resultant field expressions is obtained. Further, the magnitudes of $\mathbf{E}$ and $\mathbf{H}$, as computed from Eq. (21), are functions of the coefficients of the tensor dielectric constant $[\boldsymbol{\varepsilon}_b]$ as given by Eqs. (13) and (14). As mentioned previously, $[\boldsymbol{\varepsilon}_b]$ depends on the beam temperature, axial velocity, axial magnetic field $\mathbf{B}_0$, collision frequency, and the density of electrons. The fluctuations in such parameters will affect the resultant calculations of the electromagnetic field.

Although the analysis given previously considered TE modes, similar expressions for $\mathbf{E}$ and $\mathbf{H}$ can be obtained in case of TM modes if the same procedure is followed.

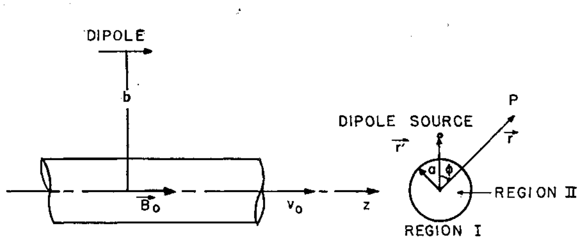

Fig. 2 Exhaust beam and dipole antenna configuration under an axial magnetic field $\mathbf{B}_0$.

III. Effect of Exhaust Beam Temperature

The interaction of the engine exhaust beam (and especially in the case of the electron beam) with the surrounding medium in a magnetic field, can result in changes in the electromagnetic field values as given by Eq. (21). The medium surrounding the beam may be represented as a stationary plasma medium. The scattering of the beam within that medium can be considered as a diffusion process. Shapiro et al.[9,10] have shown that this diffusion process is accompanied by an anisotropic change in the beam temperature in case of $B_z \neq 0$ and an isotropic change in case $B_z = 0$. Following the analysis of Refs. 9 and 10, and assuming a Maxwellian distribution function in the solution of the Boltzman-Maxwell set of equations, the changes in the longitudinal and transverse temperatures of the beam are given by Shapiro[9,10] to read

$$\frac{(\delta T_b)_{\parallel}}{T_b} = \frac{1}{140}\cdot\frac{\omega_p'^3}{N_bv_0^3}\cdot\frac{T_s}{T_b}\left(\frac{N_b}{N_s}\right)^{2/3}\cdot\tau^{-3/2}\cdot\exp\left[(3)^{1/2}\tau\cdot\left(\frac{N_b}{2N_s}\right)^{1/3}\right] \tag{22}$$

$$\frac{(\delta T_b)_\perp}{T_b} = \frac{3(2)^{1/2}}{16\pi} \cdot \frac{\omega_H^3}{N_b v_0^3} \cdot \frac{T_s}{T_b} \cdot \frac{1}{\tau} \cdot \exp\left[\left(\frac{4}{27}\right)^{1/4} \cdot \left(\frac{N_b \omega_p'}{N_s \omega_H}\right)^{1/2} \cdot \tau\right] \quad (23)$$

where

T_b = equilibrium temperature of the beam
T_s = equilibrium temperature of the medium surrounding the beam
τ = $\omega_p' t$
ω_p' = plasma frequency of the surrounding medium
ω_H = cyclotron frequency of the beam medium
N_b = density of the beam medium
N_s = density of the medium surrounding the beam
v_0 = initial z directed velocity of the beam

It is clear that the axial magnetic field $\mathbf{B}_0$ causes the anisotropy noticed in the change of the beam temperature. When $B_z = 0$, $(\delta T_b)_\perp = 0$, and the change in the beam temperature becomes isotropic. As stated before, the coefficients of the exhaust dielectric constant $[\boldsymbol{\varepsilon}_b]$ as given by Eq. (13) are complex functions of the temperature. Stix[16] has given detailed expressions for these coefficients and their corresponding dependence on the temperature. Since these expressions are given in detail in Refs. 15 and 16, they will not be included in this paper.

From the formulas given by Stix[16] and Allis[15] for the coefficients of $[\boldsymbol{\varepsilon}_b]$ as a function of the temperature and Eqs. (13, 22, and 23), the corresponding fluctuations in these coefficients due to the exhaust diffusion within the surrounding medium can be obtained. If these fluctuations in $[\boldsymbol{\varepsilon}_b]$ are substituted in Eq. (21) for the general case of $B_z \neq 0$, the resultant effect of the beam temperature change on the **E** and **H** fields is obtained. Computation of the results is very complex, and a digital computer has to be used in this case. From the resultant values of the **E** and **H** fields, the effect of the ion engine exhaust on the radiation pattern of the dipole antenna can be obtained.

It should be mentioned here also that a change in the density of the electrons is expected also during the interaction of the beam with the surrounding medium. In addition, new oscillations may be excited. The nature and growth of such oscillations has been a matter of extensive consideration in the literature, e.g., the work by Singhaus,[18] Neufeld et al.,[19] Shapiro,[9, 10] Tsyovich,[20] Lundgren et al.,[21] Stix,[16] and Akhiezer et al.[22] If all such factors are included in the analysis, the resultant expressions of the electromagnetic field of the dipole antenna become very complex for numerical evaluation. Such factors are to be considered in more detail in a future paper by the author. The initial results of this study show that these factors can affect the resultant radiation pattern of the antenna.

Conclusions

The following results are obtained from the study regarding the effect of the ion engine exhaust on the propagation of the electromagnetic waves originating from a nearby dipole antenna: 1) the resultant expressions of the **E** and **H** fields depend on the dielectric constant $[\boldsymbol{\varepsilon}_b]$ of the exhaust beam; 2) in case there is an axial magnetic field of finite value, any fluctuations in the coefficients of $[\boldsymbol{\varepsilon}_b]$ will affect the resultant values of the **E** and **H** fields; and 3) the change in the beam temperature due to its interaction with the surrounding medium also can affect the field.

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AUGUST 1965 AIAA JOURNAL VOL. 3, NO. 8

Transport Properties of Hydrogen

Dieter Brezing*
Columbia University, New York, N. Y.

The transport properties of hydrogen are calculated for a pressure range from 10^{-6} to 10^2 atm and at temperatures up to 10^6 °K. The lower temperature limit is taken where the equilibrium gas mixture contains less than 1% molecular hydrogen. The Boltzmann formalism, which assumes binary collisions in the expressions for the transport properties of gas mixtures, is applied. Theoretical values, supported by experimental data whenever possible, are used for the atom-atom, atom-ion, and atom-electron collision cross sections. For Coulomb force interactions between charged particles, the Debye length is used as cutoff distance. The collision integrals for all interactions are computed. Values of viscosity, thermal, and electrical transport coefficients, as well as the thermoelectric coefficients, are presented.

I. Introduction

During recent years the properties of a dissociating and ionizing hydrogen gas have aroused increasing interest, mainly in connection with thermonuclear research, electric propulsion projects, astrophysics, and research in plasma physics. The calculation of transport properties requires knowledge of the interaction potential between the gas particles or their collision cross sections. For molecules, atoms, and ions, these quantities have often been derived by theory and are reasonably verified by experiment. Also, the interparticle forces in a gas consisting solely of charged particles pose no problem. The missing link is constituted by interactions between atoms and electrons in the low energy region. Consequently, the region of dissociation has been under thorough exploration by various investigators.[1–3]

In the present paper, transport properties are calculated in a range where the composition of the gas changes from pure atomic hydrogen to a mixture of neutral particles, ions, and free electrons, and finally to a completely ionized gas.

For the computation of the transport properties, the kinetic theory of gas mixtures is used, which is extensively developed. It has been shown by Grad,[4] Eastlund,[5] and DeVoto[6] that the multicomponent transport theory, as presented by Hirschfelder[7] and Chapman and Cowling,[8] employing only binary collision terms is also valid for the case where charged particles are present. For plasma transport properties, the binary collision and Fokker-Planck formulations yield identical results.

The composition of the gas was calculated with the chemical equilibrium constants[9–11] using the Newton-Raphson iteration method. Electron capture was found to be unimportant in the pressure range under consideration.[12] Excited states of atomic hydrogen were neglected in this investigation, although their collision cross sections are considerably larger than those of the ground state. The occupation numbers of excited states were calculated with the approximation given by Fermi[13] using a cutoff quantum number of 10. It was found that the occupation number of the first or a higher excited state is in the order of 10^{-5} or less. This result agrees with data given by Marlow[14] and Oppenheim.[15] Thus, excited states of hydrogen contribute a negligible amount to the transport properties in the range of conditions studied here.

Received September 30, 1964; revision received April 27, 1965. This work was supported by the Air Force Office of Scientific Research under Contract No. AF 49(638)-1254. I wish to acknowledge my indebtedness to R. A. Gross for suggesting this problem, his continuous interest in this work, and his invaluable guidance and encouragement. I also thank B. J. Eastlund for his competent advice throughout this report, and J. Clearman for writing the computer program. I am indebted to NASA and the European Space Research Organization, who jointly granted my International University Fellowship.

* Diplom-Ingenieur, NASA International University Fellow. Associate Member AIAA.

II. Kinetic Theory and Transport Coefficients

A. Boltzmann Equation

The calculations of the transport properties are based upon the solution of the Boltzmann integro differential equation, i.e., knowledge of the distribution function taking into account all forces and collisions. In Chapman and Cowling[8] and Hirschfelder[7] detailed derivations can be found; the notation of the latter will be used in this report. The result is that all transport properties can be expressed in terms of collision integrals, which are independent of pressure and functions of temperature only.

The transport properties are expressed in the following equations:

$$j = \sigma E + \alpha \nabla T \tag{1}$$

$$q = -\beta E - \mathrm{k} \nabla T \tag{2}$$

$$p = pU - 2\eta S \tag{3}$$

where j denotes the current, q the heat flux, and p the pres-